Electric Permittivity and Magnetic Permeability

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In our studies of electricity and magnetism, the ubiquitous constants ε_o and μ_o, respectively called the "electric permittivity" and "magnetic permeability" of vacuum, set the scale for electric and magnetic phenomena. The usage and meaning of these constants raises some curious questions. For instance, why would someone choose to define them in a way that spoils the otherwise beautiful symmetry between analogous formulas for electric and magnetic quantities? To illustrate, we find the energy density of the electric field **E** to be $\frac{1}{2}\varepsilon_o E^2$, but the energy density of a magnetic field **B** is $\frac{1}{2}B^2/\mu_o$ instead of $\frac{1}{2}\mu_o B^2$. Furthermore, how do their values conspire to give the speed of light? And given that the "sub-0" denotes the properties of vacuum, if the vacuum is nothing but empty space, how can any number other than zero characterize its properties?[1] Why is 4π usually connected with ε_o and μ_o? Why was the value of $1/4\pi\varepsilon_o$ historically measured to several significant figures, while the value of $\mu_o/4\pi$ was said to be exact?

These constants seem to slip in through the back door. In classical field theory we meet the electrostatic field $\mathbf{E}(\mathbf{r})$ and the magnetostatic field $\mathbf{B}(\mathbf{r})$. The electric field at $\mathbf{r}$, due to a point source charge q' located at $\mathbf{r}'$, is given by Coulomb's law,

$$\mathbf{E}(\mathbf{r}) = k_e q'\mathbf{R}/|\mathbf{R}|^3 , \qquad (1)$$

where the vector $\mathbf{R} \equiv \mathbf{r} - \mathbf{r}'$ goes from the source point to the field point (see Fig. 1a).

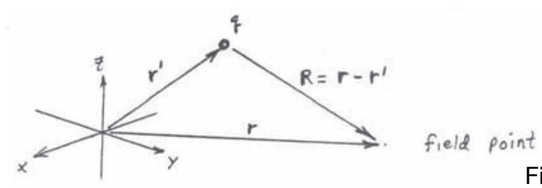

Figure a

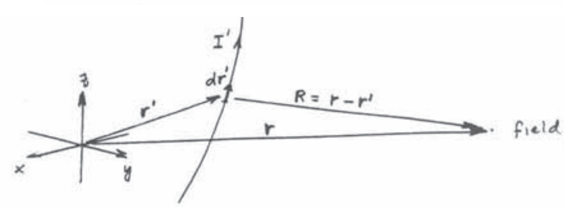

Figure b

Fig. 1. Geometry of the laws of Coulomb (a) and Biot–Savart (b).

Magnetic fields are produced by electric currents. A static magnetic field requires a steady flow of charges, a DC current I'. Let $d\mathbf{r}'$ denote the infinitesimal displacement vector of one of those charges instantaneously at a fixed location (tangent to conventional current). The infinitesimal piece of directed current $I'd\mathbf{r}'$ serves as a point source and contributes the increment of magnetic field $d\mathbf{B}(\mathbf{r})$ at the field point according to the Biot–Savart law,

$$d\mathbf{B}(\mathbf{r}) = k_m I' \, d\mathbf{r}' \times \mathbf{R}/R^3. \qquad (2)$$

This must be integrated around the entire circuit to yield the **B** field at **r**.

The values of k_e and k_m set the scale for these fields and the forces they produce. In SI units their values are

$$k_e = 8.988 \times 10^9 \text{ Nm}^2/\text{C}^2 \qquad (3)$$

to four significant figures, and

$$k_m = 1 \times 10^{-7} \text{ Ns}^2/\text{C}^2 \qquad (4)$$

exactly. As you might surmise, k_m was historically *defined* and k_e was *measured.*

When we derive Gauss's law for **E** from Coulomb's law (take the scalar product of Eq. (1) with $\boldsymbol{n}dA$, a patch of area dA with outward-pointing unit vector $\boldsymbol{n}$, and integrate over a closed surface), we pick up a factor of 4π from a solid angle and obtain

$$\oint \mathbf{E} \cdot \mathbf{n} \, dA = 4\pi \, k_e q_{enclosed} , \qquad (5)$$

where the charge $q_{enclosed}$ resides within the closed surface. In differential form,[2] Gauss' law says

$$\nabla \cdot \mathbf{E} = 4\pi k_e \rho, \qquad (6)$$

where ρ denotes the charge density. In deriving Ampère's law from the Biot–Savart law, another 4π appears and Ampère's law gives

$$\oint \mathbf{B} \cdot d\mathbf{r} = 4\pi k_m I_{pierce}, \qquad (7)$$

where I_{pierce} denotes the current that pierces any surface bounded by the closed contour. In differential form Ampère's law becomes[3]

$$\nabla \times \mathbf{B} = 4\pi k_m \mathbf{j}, \qquad (8)$$

where **j** denotes the electric current density.

It is customary to hide the ubiquitous 4π's by dividing them out of the Coulomb and Biot–Savart constants, thereby introducing ε_o and μ_o,

$$k_e \equiv 1/4\pi\varepsilon_o \quad \text{and} \quad k_m \equiv \mu_o/4\pi, \qquad (9)$$

the so-called "rationalized" units of Oliver Heaviside.[4] From Eqs. (3), (4), and (9) we obtain

$$\varepsilon_o = 8.844 \times 10^{-12} \text{ C}^2/\text{Nm}^2$$

and

$$\mu_o = 4\pi \times 10^{-7} \text{ Ns}^2/\text{C}^2. \qquad (10)$$

With them the laws of Gauss and Ampère look less cluttered:

$$\nabla \cdot \mathbf{E} = \rho/\varepsilon_o \qquad (11)$$

and

$$\nabla\times\mathbf{B} = \mu_o\,\mathbf{j}. \qquad (12)$$

There is nothing fundamental about choosing ε_o and μ_o over k_e and k_m ; the choice is a matter of taste and convenience.

The constant ε_o is called the "electric permittivity" and μ_o the "magnetic permeability" of *vacuum*. The permittivity and permeability of *matter* take on values that are rescalings of ε_o and μ_o. These names suggest that the permittivity measures in some sense the transparency of a medium to an electric field, and permeability the ability of the medium to support a magnetic field. The rescaling coefficients that distinguish matter from vacuum have names such as *susceptibilities*, *dielectric constants*, and the *index of refraction*. First, let us see why μ_o was originally defined and ε_o was originally measured.

The Values of ε_o and μ_o

In any system of physical measurement, some units must be defined as standards. For instance, to measure length one could choose a reference body and then express all other lengths as multiples of it. An amusing instance occurred in 1958 when some MIT students measured the length of a bridge that spans the Charles River by using their classmate Oliver Smoot as the unit. The length of the bridge was reported to be 364.4 smoots, plus or minus one ear.[5] Since the smoot as a unit of length is not reproducible in all places for all time, a less subjective standard is needed. The SI system (Système International) that you met in General Physics originally defined units for length (meter, m), mass (kilogram, kg), and time (second, s). The second is today defined in terms of the frequency of a certain spectral line of cesium-133. Before 1960 the meter was originally defined by a specific archival bar. Between 1960 and 1983 the meter was defined in terms of a wavelength of a spectral line emitted by krypton-86 (more about 1983 later). Mass was and still is (for now) defined in terms of an archived body, the carefully preserved platinum-iridium standard kilogram, "Le Grande K" stored near Paris, France.[6] All dimensioned observables in the SI or "mks" system reduce to some combination of meter, kilograms, and seconds. For example, the unit of force (the Newton, N) is kg-m/s^2.

A close relative of the mks system is the "cgs" system, which uses centimeters (cm) for length (1 m = 100 cm), grams (g) for mass (1 kg = 1000 g), and the second for time. Here the unit of force is the dyne (1 N = 10^5 dynes). If you routinely work with small amounts of material, then cgs may be more convenient than mks.

Electric charge and current can be defined in terms of force. Historically, there were two ways to do this:[7] (1) an electrostatic definition of charge, where we choose a value for ε_o and then measure the force between two standardized charge configurations; or (2) a magnetostatic definition of current, where a value for μ_o is chosen and the force measured between two standardized currents. Let us see how this works in both instances.

Since a point charge q' produces a field $\mathbf{E}$ given by Coulomb's law, and another point charge q "feels" the force $\mathbf{F} = q\mathbf{E}$, the magnitude of the electrostatic force $\mathbf{F}$ between two identical point charges will be

$$F = (1/4\pi\varepsilon_o)\, q^2/R^2 \qquad (13)$$

so that, by dimensional analysis, $q \sim \sqrt{(F\varepsilon_o)} \times (length)$. The cgs system gives the electric charge unit the name "electrostatic unit" or esu or "statcoulomb," and defines $1/4\pi\varepsilon_o$ = 1 dyne-cm^2/esu^2. By definition, one esu of charge is carried by each of two identical point charges so that when 1 cm apart, the electric force between them equals 1 dyne. If you have done a Cavendish balance lab, then you have been through a version of this procedure. With charge defined, electric current, $I = dq/dt$, can also be defined in these units: 1 "statampere" = 1 esu/s.

The SI system historically used the second alternative (the procedure described below pertains to definitions of charge and current before 1983). For instance, at the distance r from an infinitely long straight wire, the Biot–Savart or Ampère laws gives $B = \mu_o I/2\pi r$. If another identical wire parallel to the first also carries current I, a segment of it having length l feels the force magnitude $F = IlB$, which we rearrange as

$$F/l = \mu_o I^2/2\pi r. \qquad (14)$$

The unit of current that we call the Ampère (A) is defined to be the current such that, with these wires 1 m apart, a force per length of 2×10^{-7} N/m is produced. The size of the "amp" is determined by the value chosen for μ_o. In the SI system, we *define* $\mu_o = 4\pi\times10^{-7}$ N/A^2. The unit of charge, the Coulomb, is defined as 1 C = 1 A-s. When 1 A of current flows by you for 1 s, then a total charge of 1 Coulomb (C) has passed by. It takes a lot of elementary particles to make a Coulomb—the charge of one electron is 1.6×10^{-19} C.

So far we have values for μ_o and ε_o but in different units. But notice that

$$\mu_o\varepsilon_o = 10^{-2}\,(1\text{ esu/C})^2\,(1\text{ cm/s})^{-2}. \qquad (15)$$

To find the esu-to-Coulomb ratio, go back to either experiment and put into it a known amount of charge or current measured from the other system's definition. For example, in the two-charge system used to define the esu, replace the 1 esu with a known number X of Coulombs, keep the point charges 1 cm apart, and remeasure the force in dynes. Taking the ratio of forces, the $1/4\pi\varepsilon_o$ drops out, leaving

$$F_2/F_1 = (X\text{ C}/1\text{ esu})^2. \qquad (16)$$

By measuring X and the force ratio (rounding the measured 2.9979250 to 3 in this discussion), one finds that 1 C = 3×10^9 esu, and Eq. (15) becomes $\mu_o\varepsilon_o = 1/(3\times10^{10}\text{ cm/s})^2$.[8] It will not escape notice that

$$\mu_o\varepsilon_o = 1/c^2 \qquad (17)$$

where c denotes the pre-1983 *measured* speed of light in vacuum!

One might wonder if someone slyly worked backward from the measured value of c to engineer the value of μ_o that ensures Eq. (17). Suppose we try it! If the value of μ_o had been chosen to have some other value, say rescaled by a factor S from the choice mentioned above, so that our μ_o gets replaced with $\mu_o \rightarrow S\mu_o$, then the Ampère unit, and likewise the Coulomb, would have been rescaled by $1/\sqrt{S}$. That means there would be a compensating S on the right-hand side of Eq. (15), which would cancel the S in $\mu_o \rightarrow S\mu_o$ on the left side of that equation. This seeming coincidence between electrostatic and electromagnetic units and the speed of light suggests a deep insight: Light is electromagnetic! The mechanism of that linkage would have to await the rest of Maxwell's equations, with their prediction of waves in the electromagnetic field that propagate in vacuum at the speed $1/\sqrt{(\mu_o\varepsilon_o)}$.

George Gamow has reflected, "The numerical coincidences between seemingly unconnected physical quantities, such as the ratio of electrostatic and electromagnetic units on the one side, and the velocity of light on the other, often led to fundamental new discoveries and broad generalizations in physics."[9] The light-as-electrodynamics case is not unique, as Gamow reminds us. For instance, the seeming coincidence between the constant that Max Planck used in 1900 to fit the spectrum of blackbody radiation and the constant that Einstein used in 1905 to fit the energy spectrum of electrons in the photoelectric effect triggered the development of quantum mechanics. The equivalence of gravitational and inertial mass offers another instance where an apparent "coincidence" led, on deeper inspection, to profound insight into the unification of gravitation and energy with space and time.

Now we can put ε_o into the same units as μ_o, from Eq. (17) and using the measurements described above. We find (replacing 3 with the present standard value 2.99792458)[10]

$$\varepsilon_o = 8.854187817\times10^{-12}\ \mathrm{C^2/Nm^2}. \qquad (18)$$

These considerations describe how electric charge and current were defined before 1983. With the development of high-speed electronics, it became possible by 1983 to redefine the meter in terms of the distance that light travels in a tiny fraction of a second. In particular, the speed of light is now *defined* to be 299,792,458 m/s, which was the same as defining the meter to be the distance light travels in 1/299,792,458 s.[10] With both μ_o and the speed of light defined, from Eq. (17) we can say that, since 1983, ε_o no longer needs to be measured. However, I find it astonishing that the insight "light is electromagnetic" came from *static* electric and magnetic measurements. This was done by measuring two of the three quantities in the trio μ_o, ε_o, and c, which were then put together with the discovery of the relation $\mu_o\varepsilon_o = 1/c^2$. That insight occurred long before high-speed electronics existed.

The finite value of the speed of light suggests that the vacuum of empty space offers impedance to the flow of electromagnetic energy. As the names of ε_o and μ_o suggest, such impedance even for vacuum will be related to these constants. The impedance Z that any medium presents to a current I sent through it is related to the voltage V driving that current in a generalization of Ohm's law:

$$Z = V/I. \qquad (19)$$

By definition, voltage is the line integral of **E**. By Ampère's law (generalized to include Maxwell's displacement current, see Eq. (22) below), the current is $1/\mu_o$ times the line integral of the magnetic field. The geometrical factors in the line integrals cancel, giving us $Z = \mu_o E/B$. Looking ahead to other parts of electrodynamics, we note that the amplitudes of harmonic *radiation* fields (light waves) are related in SI units by $E = cB$, so that, for space filled with light waves but no matter, we find that vacuum offers to the propagation of light waves the impedance

$$Z = \mu_o c = \sqrt{\mu_o/\varepsilon_o} \approx 377\ \text{ohms}. \qquad (20)$$

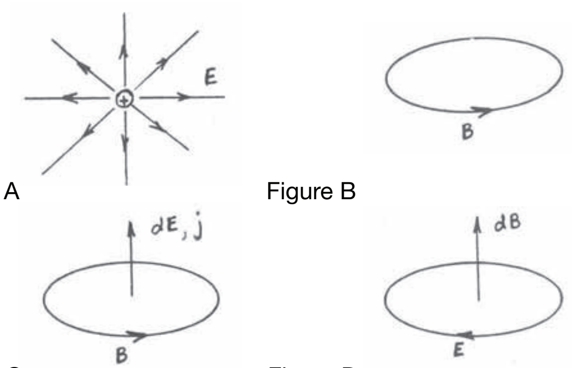

Figure A Figure B Figure C Figure D

Fig. 2. Cartoon illustrations of Maxwell's equations: (A) Gauss's law for **E**; (B) Gauss's law for **B**; (C) the Ampere–Maxwell law; and (D) Faraday's law.

Let us go beyond vacuum and look at the permittivity and permeability in matter. These quantities depart from their vacuum values because externally applied electric and magnetic fields impinging on matter can distort the molecules, producing or enhancing electric and magnetic dipole moments. These dipoles then produce electric and magnetic fields of their own, which combine with the original external field.

Before going there, let us write the time-dependent Maxwell equations that hold in vacuum *or* in matter (see Fig. 2).[4] One is Gauss's law for **E**, Eq. (11), which says **E** field streamlines diverge from (or converge toward) their source charges. Gauss's law for **B** conveys a similar relationship, although physics has yet to find evidence for isolated magnetic poles—the density of magnetic monopoles evidently vanishes:

$$\nabla\cdot\mathbf{B} = 0. \qquad (21)$$

Streamlines of **B** do not diverge from or converge to a point, but can only close back on themselves.

Moving charges and changing electric fields produce **B** fields with whirlpools (a "curl"), described by the Ampère–Maxwell law, generalizing Eq. (12) to

$$\nabla\times\mathbf{B} = \mu_o\,(\,\mathbf{j} + \varepsilon_o\,\partial\mathbf{E}/\partial t\,) \qquad (22)$$

where t denotes time.

When the magnetic field is time-dependent, it generates an electric field with whirlpools, and Faraday's law says

$$\nabla\times\mathbf{E} = -\partial\mathbf{B}/\partial t. \qquad (23)$$

In all these expressions, the charge density ρ and the current density **j** are totals due to *all* charged particles in the system. It is crucial in what follows to distinguish total charges and currents from the so-called "free charges" and "mobile currents." Free charges are those *in excess* of those that make up neutral matter. A prototypical example would be the excess positives in one plate (and the same number of excess negatives on the other plate) of a charged capacitor. They set up an electric field between the plates, which points from the free positives to the free negatives. When we insert our sample material between the plates, the molecules are "hit" by this original electric field.

Mobile currents are merely charged particles in motion, such as the loosely bound outer-shell electrons in the atoms of a good conductor. To visualize a venue for mobile currents, imagine the current flowing in the wire of a solenoid. When our sample of matter is placed inside the solenoid, its atoms are hit with its original magnetic field.

These externally applied electric and magnetic fields may induce electric and magnetic dipole moments among the material's molecules. Consider the electric case. When the capacitor's electric field is switched on, in the material between the plates a molecule's electrons are pulled away from the negative plate of the capacitor and toward the positive plate, while the positive charges are oppositely pulled. The molecules are still electrically neutral, but they get "stretched" with the centers of positive and negative charge separated. The molecules have become little dipoles, and each one produces its own electric field. That field's direction opposes the original one that was set up between the capacitor plates by the free charges.

An idealized electric dipole consists of equal and opposite charges, $+q$ and $-q$, separated by some small distance. Recall that a point charge q sets up an electric potential given by

$$V(\mathbf{r}) = (1/4\pi\varepsilon_o)\, q/R \qquad (24)$$

where $R \equiv |\mathbf{r} - \mathbf{r}'|$ denotes the distance from the source point to the field point. The potential of a dipole will be the superposition of such terms for both charges. Being a pair of charges, the electric potential produced by an electric dipole is (to lowest order in R),

$$V(\mathbf{r}) = (1/4\pi\varepsilon_o)\, \mathbf{p}\cdot\mathbf{R}/R^3 \qquad (25)$$

where the electric dipole moment $\mathbf{p}$ is defined as $\mathbf{p} \equiv q\mathbf{a}$, with $\mathbf{a}$ the displacement vector from $-q$ to $+q$.

Consider now a sample of matter that consists of polar molecules, and let $\mathbf{P} = \mathbf{P}(\mathbf{r}')$ denote the density of electric dipole moments. The potential due to a distribution of such dipoles follows by superposition of Eq. (25) over the volume of the matter:

$$V(\mathbf{r}) = (1/4\pi\varepsilon_o) \int (\mathbf{P}\cdot\mathbf{R})/R^3\, dx'dy'dz'$$

$$= (1/4\pi\varepsilon_o) \int \mathbf{P}\cdot\nabla'(1/R)\, dx'dy'dz' \qquad (26)$$

where the gradient operator ∇' takes derivatives with respect to the $\mathbf{r}'$ coordinates. By integrating by parts and using Gauss's divergence theorem on one of the terms,[2] we obtain one contribution to V from the surface of the material and another from the bulk volume:[11]

$$V(\mathbf{r}) = (1/4\pi\varepsilon_o) \Big[\oint (\mathbf{P}\cdot\mathbf{n})/R\, dA'$$

$$- \int (\nabla'\cdot \mathbf{P})/R\;\, dx'dy'dz' \Big]. \qquad (27)$$

By comparing these results to the generic superposition based on Eq. (24), in particular[12]

$$V(\mathbf{r}) = (1/4\pi\varepsilon_o) \int \frac{\rho(\mathbf{r}')}{R}\, dx'dy'dz', \qquad (28)$$

we identify $\mathbf{P}\cdot\mathbf{n}$ as a charge per unit area due to polarization on the surface of the material, and $-\nabla\cdot\mathbf{P}(\mathbf{r})$ as the volume density of polarization charges (now dropping the prime because we are henceforth looking at the $\mathbf{P}$ field itself, not having to distinguish source points from field points).

Returning to Gauss's law for **E,** Eq. (11), we now see that the total charge density can be split into a contribution of free charges and another one from the electric dipoles:

$$\nabla\cdot\mathbf{E} = (\rho_{free} - \nabla\cdot\mathbf{P}) / \varepsilon_o. \qquad (29)$$

This can be transposed to appear as

$$\nabla\cdot\mathbf{D} = \rho_{free} \qquad (30)$$

where

$$\mathbf{D} \equiv \varepsilon_o\mathbf{E} + \mathbf{P}. \qquad (31)$$

When no matter exists other than the free charges, then $\mathbf{P} = \mathbf{0}$; **D** and **E** are two names for the same field, except for the factor of ε_o. But with polarization "turned on," **D** and **E** have different roles: **D** is the electric field due to free charges and **E** the total electric field. Polarization charges orient themselves so their field partially cancels the **D** field of the free charges; thus, **E** is proportional to $\mathbf{D} - \mathbf{P}$.

Before we turn to magnetic dipoles, we should notice that the polarization charge density, when time dependent, forms an electric current that must be included among magnetic field sources. A surface layer of polarized material has charge per area $\mathbf{P}\cdot\mathbf{n}$ which, if changing (imagine time-dependent molecular stretching), means a current through the layer exists, given by

$$I_{pol} = \int (\partial\mathbf{P}/\partial t)\cdot\mathbf{n}\, dA \qquad (32)$$

and thus

$$\mathbf{j}_{pol} = \partial\mathbf{P}/\partial t. \qquad (33)$$

Turning to magnetism, because the divergence of a curl identically vanishes, Gauss's law for **B** says that **B** may be written as the curl of a vector potential **A**. For a generic current density **j**, **A** is given by [12]

$$\mathbf{A}(\mathrm{r}) = (\mu_o/4\pi) \int \frac{\mathbf{j}(\mathbf{r}')}{\mathrm{R}}\, dx'dy'dz'. \qquad (34)$$

From this one can show that the **A** of a magnetic dipole of moment **m** is[13]

$$\mathbf{A} = (\mu_o/4\pi)\, \mathbf{m}\times\mathbf{R}/\mathrm{R}^3. \qquad (35)$$

In the presence of an external magnetic field the molecules in matter may acquire a magnetic dipole moment **m** or have an existing one enhanced. Consider a chunk of matter that carries magnetic dipole moment per unit volume **M**. By superposition, and with the same tricks (integration by parts) that we used in the electric dipole case, you can show that $\nabla\times\mathbf{M}$ forms a current density due to the magnetic dipoles.

Now the Ampère–Maxwell law, Eq. (22), may be written with mobile currents distinguished from electric and magnetic polarization currents. Explicitly, with $\mathbf{j}_{total} = \mathbf{j}_{mobile} + \mathbf{j}_{pol} + \mathbf{j}_{mag}$, Eq. (22) becomes

$$\nabla\times\mathbf{B} = \mu_o(\mathbf{j}_{mobile} + \partial\mathbf{P}/\partial t + \nabla\times\mathbf{M} + \varepsilon_o\partial\mathbf{E}/\partial t). \quad (36)$$

Now use Eq. (31) and transpose the result to write Eq. (36) as

$$\nabla\times\mathbf{H} = \mathbf{j}_{\text{mobile}} \qquad (37)$$

where

$$\mathbf{H} \equiv \mathbf{B}/\mu_o - \mathbf{M}. \qquad (38)$$

Notice that the total magnetic field **B** is proportional to **H** + **M**; the magnetism induced in the sample typically enhances the externally applied field.

We have rewritten the Maxwell equations with source terms to distinguish "free and mobile" sources from polarization sources. The remaining two Maxwell equations have no source terms, so they can remain expressed in terms of **E** and **B.** When a charge q finds itself in the presence of electric and magnetic fields, the electromagnetic force on it is still $q(\mathbf{E} + \mathbf{v}\times\mathbf{B})$: the *total* fields exert force on a test charge.

Constitutive Relations

Now we need constitutive relations that relate **P** and **M** to the other fields. For an isotropic medium we define its dimensionless "electric susceptibility" χ according to

$$\mathbf{P} = \varepsilon_o\chi\mathbf{E}. \qquad (39)$$

If χ is not itself a function of **E** then the material is said to be linear (when χ *does* depend on **E,** then we are in the regime of nonlinear optics). From Eq. (31) we obtain the constitutive relation between **D** and **E**,

$$\mathbf{D} = \varepsilon_o(1 + \chi)\mathbf{E} \equiv \varepsilon\mathbf{E}. \qquad (40)$$

The factor $\kappa \equiv 1+\chi$ is called the "dielectric constant" and $\varepsilon_o\kappa \equiv \varepsilon$ the "permittivity" of the material.

Some materials, when hit with an electric field **E** in, say, the x-direction, may show a polarization in, say, the y-direction. To allow for such cases we define the *susceptibility tensor* (or matrix), a quantity with nine components $\{\chi_{\mu\nu}\}$, according to $P_i = \varepsilon_o \chi_{ij} E_j$. The subscripts denote the various components, and repeated indices are summed over all three of them. From here one can go on to define dielectric and permittivity tensors.

In a similar way the magnetic susceptibility ψ is defined for an isotropic medium as $\mathbf{M} = \psi\mathbf{H}$**,** which, by Eq. (38), also gives the constitutive relation between **B** and **H,**

$$\mathbf{B} = \mu_o(1 + \psi)\mathbf{H} \equiv \mu\mathbf{H} \qquad (41)$$

where $\mu \equiv \mu_o(1+\psi)$ denotes the "permeability" of the medium. Generalizing to tensor relations for nonisotropic materials is straightforward.

The speed of light v in a medium is related to its permittivity and permeability according to $\mu\varepsilon = 1/v^2$. The index of refraction n of a piece of material is determined by the polarizability of its molecules, according to[14]

$$n = c/v = [\mu\varepsilon/\mu_o\varepsilon_o]^{1/2} = [(1+\psi)(1+\chi)]^{1/2}. \qquad (42)$$

Such macro/micro connections generalize, of course. The electric and magnetic susceptibilities, and other such coefficients, can be predicted by using statistical mechanics applied to models of the molecules that make up the material. These constants offer a window from the macroscopic world of voltmeters and ammeters into the microscopic world of atoms and molecules.

Finally, you might wonder why the definitions put $k_m \sim \mu_o$ but $k_e \sim 1/\varepsilon_o$ instead of the more symmetrical $k_e \sim \varepsilon_o$. I suppose this is due to the lack of symmetry in the definitions of capacitance and inductance. In terms of self-inductance L and capacitance C, the voltage across an inductor is Ld^2q/dt^2 but is q/C for the capacitor. Notice the inverse relation between voltage and charge (or its derivative) in comparing C to L. That inversion explains, I think, why the permittivity and permeability are defined according $k_m \sim \mu_o$ but $k_e \sim 1/\varepsilon_o$. This way, whenever you derive a formula for an object's capacitance you get $C \sim \varepsilon_o\times$ (*length*), and for inductance you get $L \sim \mu_o\times$(*length*). The SI units of ε_o may be written as Farad/meter (F/m) and for μ_o they are Henry/meter (H/m). "Farads" and "Henrys" are the SI names for the units of capacitance (1 F = 1volt/Coulomb) and inductance (1 H = 1 volt/(C/s^2)).

From vacuum to superconductors, from water to iron, a body's permittivity and its permeability tell us complex stories about the medium's interactions with electric and magnetic fields.

Acknowledgment

I thank Thomas Olsen for carefully reading a draft of the manuscript and making many helpful suggestions.

References

[1] The vacuum, itself, is a curious object of study—think of dark energy, for example, in cosmology.

[2] We make much use of Gauss's divergence theorem, which says that the volume integral of the divergence of a vector field V equals the flux of V through the surface enclosing the volume:

$$\iiint \nabla\cdot\mathbf{V}\,dx\,dy\,dz = \oiint \mathbf{V}\cdot\mathbf{n}\,dA$$

where dA is a patch of area on the surface and **n** an outward-pointing normal unit vector.

[3] We also need Stokes' theorem, which says that the flux through a surface of the curl of a vector field V equals the line integral of V around the perimeter of the surface:

$$\iint (\nabla\times\mathbf{V})\cdot\mathbf{n}dA = \oint \mathbf{V}\cdot d\mathbf{r}$$

[4] Maxwell's equations and constitutive relations as Maxwell wrote them spelled out, component by component, some 20 equations. Oliver Heaviside developed vector calculus and put Maxwell's equations into the concise form that we use today.

[5] *http://en.wikipedia.org/wiki/Smoot*. This source notes that, fittingly, Oliver Smoot later became the chairman of the American National Standards Institute and president of the International Organization for Standardization.

[6] Quantum effects, notably the Josephson junction and the quantum Hall effect, may make the reliance on Le Grande K unnecessary, analogous to how special relativity with its invariance of the speed of light made unnecessary an archival body as the standard of length. See Neil M. Zimmerman, "Quantum electrical standards," *Physics Today,* Aug 2010, p. 68.

[7] Well described in E.U. Condon and H. Odishaw, Eds., *Handbook of Physics* (McGraw-Hill, 1958), p. 9 of the "Units and Conversion Factors" section.

[8] E.g., see J.D. Jackson, *Classical Electrodynamics* (Wiley, 1975), p. 812; and D. Griffiths, *Introduction to Electrodynamics,* 3rd ed. (Prentice-Hall, 1999), p. 559.

[9] G. Gamow, *The Great Physicists from Galileo to Einstein* (Dover, 1988), pp. 156–157.

[10] E.g., see R. Resnick, D. Halliday, and K. Krane, *Physics,* Vol. 1, 4th ed. (Wiley, 1992), Ch. 1. The NIST value may be found at *http://physics.nist.gov/cuu/Constants/index.html*.

[11] For the details of these manipulations, see, e.g., D. Griffiths, ref. 8, Ch. 4 for electric polarization, and Ch. 6 for magnetic polarization.

[12] For brevity, the formulas for the potentials V and **A** are written as though for static sources. However, these expressions are correct in time dependent situations if we choose the Lorentz gauge, and if the time delay R/c between the doings of the source point and the resulting change in the field at the field point are taken into account.

[13] E.g., Griffiths, ref. 8, Ch. 5.

[14] For a simplified model of the "Origin of the Refractive Index" in terms of light scattering by oscillating electrons (and thus electric dipoles), see R.P. Feynman, R.B. Leighton, and M. Sands, *The Feynman Lectures on Physics* (Addison-Wesley, 1963), Ch. 31 of Vol. 1.